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Optimum shape and dimensions of ducts for convective heat transfer in laminar flow at constant wall temperature

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Abstract

One knows that a certain geometry exists for a given pressure loss in a duct, where maximum heat transfer occurs. In this work, the maximum heat transfer and the optimum geometry for a given pressure loss have been calculated for forced convective heat transfer in different duct shapes for laminar flow conditions. Simple equations which enable calculation of these optimum values for all *Pr* numbers and for all shapes of duct cross-sectional areas have been derived. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Convective heat transfer; Channel flow; Laminar flow; Constant wall temperature; Optimization

1. Introduction

Regenerators and recuperators used for energy recovery systems consist of small-diameter ducts, in which laminar flow prevails. These ducts can have different cross-sectional areas. One knows from previous investigations that an optimum spacing exists for a parallel plate channel in which forced convection [1] or natural convection [2,3] takes place. However, there exists no investigation about the optimum shape of cross-sectional areas. In this work, the optimum hydraulic diameter and the maximum heat transfer in ducts of arbitrary cross-sectional area are investigated.

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2. Derivation of equations for optimum dimensions

We consider constant wall temperature and seek the maximum heat transfer for a given pressure drop.

The heat transfer in a duct can be formulated as

$$\dot{Q} = \rho c_p \dot{V}(T_\mathrm{i} - T_\mathrm{e}) \quad (1)$$

where ρ, c_p, $\dot{V}$, T_i and T_e, respectively, are density, specific heat, volume flow rate, mean inlet temperature and mean exit temperature of the fluid.

We are interested in the heat transfer per cross-sectional area;

$$\dot{q}_A = \frac{\dot{Q}}{A} \quad (2)$$

Then it follows from Eq. (1);

$$\dot{q}_A = \rho c_p u_\mathrm{m}(T_\mathrm{i} - T_\mathrm{e}) \quad (3)$$

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Nomenclature

a	thermal diffusivity
A	cross-sectional area
c_p	specific heat
d	diameter
f	factor
h	heat transfer coefficient
k	thermal conductivity
K	pressure drop number
L	length of duct
n	number of ducts
Nu	Nusselt number
P	periphery
Pr	Prandtl number
$\dot{q}$	heat flux
$\dot{Q}$	heat flow
Re	Reynolds number
T	temperature
u	velocity
$\dot{V}$	volume flow rate
x	axial coordinate
z	dimensionless axial coordinate

Greek symbols

Δp	pressure drop
ΔT	temperature difference
Φ	heat transfer factor
ε	porosity
φ	shape factor for pressure loss
ν	kinematic viscosity
θ	dimensionless temperature
ρ	density

Superscripts and Subscripts

*	dimensionless
e	exit
f	frictional
h	hydraulic
i	inlet
l	local
m	mean
o	optimum
w	wall
∞	for $n \to \infty$ or $z \to \infty$

Here u_m is the mean fluid velocity. We define the following dimensionless numbers:

$$\theta = \frac{T_e - T_w}{\Delta T} \tag{4}$$

$$u^* = \frac{u_m}{u_p} \tag{5}$$

Here u_p and ΔT are defined as follows:

$$\Delta T = T_i - T_w \tag{6}$$

$$u_p = \sqrt{2\Delta p/\rho} \tag{7}$$

in which T_w and Δp, respectively, are wall temperature and total pressure loss in the duct.

Using the definitions above, one gets from Eq. (3):

$$q_A^* = u^*(1-\theta) \tag{8}$$

with

$$q_A^* = \frac{\dot{q}_A}{\rho c_p u_p \Delta T} \tag{9}$$

u^* and θ should be calculated using the equation for pressure loss and heat transfer in the duct, respectively.

The pressure drop in a duct Δp consists of frictional and incremental pressure loss Δp_f and local pressure loss (inlet and outlet) Δp_l:

$$\Delta p = \Delta p_f + \Delta p_l \tag{10}$$

Local pressure loss Δp_l can be calculated from:

$$\Delta p_l = K_l \frac{\rho u_m^2}{2} \tag{11}$$

K_l is to be determined using the data from White [4] and Brauer [5] according to the following relationship:

$$K_l = \frac{(3-\varepsilon)(1-\varepsilon)^2}{2-\varepsilon} \tag{12}$$

In this equation, ε is the porosity of heat exchanger ducts. ε can be envisaged as the ratio of the fluid velocity before entering the duct to that in the duct. If $\varepsilon = 0$ (similar to a duct connected between two large tanks), we have $K_l = 1.5$, and for $\varepsilon = 1$ (no contraction at the inlet and no expansion at the outlet), we have $K_l = 0$.

With the definition of dimensionless pressure loss:

$$\Delta p^* = \frac{\Delta p}{\rho u_m^2/2} \tag{13}$$

Eq. (10) yields

$$\Delta p^* = \Delta p_f^* + K_l \tag{14}$$

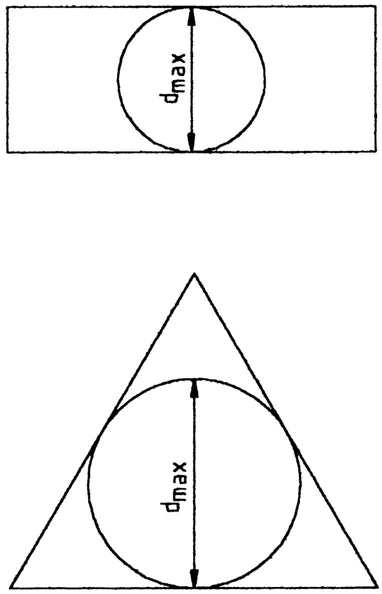

Fig. 1. Definition of d_{max}.

Pressure loss Δp_f, which includes the frictional pressure loss and the pressure loss due to velocity profile development in the developing section of the duct, can be determined for ducts of different shapes using the equation given by Yilmaz [6]:

$$\Delta p_f^* = 64\varphi x^* + \frac{13.766 x^{*0.5}}{\left[1 + 13.95\varphi x^{*0.5} + \left(\frac{13.766}{K}\right)^3 x^{*1.5}\right]^{1/3}} \tag{15}$$

where φ and K are the shape factor and the incremental pressure drop number, respectively. They can be calculated from:

$$\varphi = 1 + \frac{\varphi_\infty - 1}{1 + 0.33 d^{*2.25}/(n-1)} \tag{16}$$

$$K = \frac{1.33}{1 + (1.33/K_\infty - 1)/\left[1 + 0.74 d^{*2}/(n-1)\right]} \tag{17}$$

φ_∞ and K_∞ are given by the equations below:

$$\varphi_\infty = \frac{3}{8} d^{*2}(3 - d^*) \tag{18}$$

$$K_\infty = \frac{12}{5}(3 - d^*)^2 \left[\frac{9}{7}\frac{3 - d^*}{7 - 3d^*} - \frac{1}{5 - 2d^*}\right] \tag{19}$$

in which d^* and n are dimensionless numbers to describe the shape of the duct cross-section:

$$d^* = \frac{d_h}{d_{max}} \tag{20}$$

$$n = \frac{P}{P_h} = \frac{A}{A_h} \tag{21}$$

where d_h is the hydraulic diameter of the duct, P_h and A_h are, respectively, the periphery and the cross-sectional area of the circular duct having the hydraulic diameter d_h, and d_{max} is the maximum diameter of the circle which inscribes the actual cross-section and is shown in Fig. 1.

The dimensionless duct length x^* used in Eq. (15) is defined below:

$$x^* = \frac{L}{d_h}\frac{1}{Re} \tag{22}$$

where Reynolds number is

$$Re = \frac{u_m d_h}{\nu} \tag{23}$$

Eq. (22) can be rewritten using Eqs. (5) and (23) as

$$x^* = \frac{1}{u^* d_h^{*2}} \tag{24}$$

with

$$d_h^* = \frac{d_h}{\sqrt{\nu L/u_p}} \tag{25}$$

Substituting Eq. (15) into Eq. (14), one gets the following relationship:

$$64\varphi x^* + \frac{13.766 x^{*0.5}}{\left[1 + 13.95\varphi x^{*0.5} + \left(\frac{13.766}{K}\right)^3 x^{*1.5}\right]^{1/3}} + K_1 = \Delta p^* \tag{26}$$

Using the definitions in Eqs. (5), (7), (24) and (25), we then get Eq. (27) from Eq. (26):

$$64\varphi \frac{u^*}{d_h^{*2}} + \frac{13.766 u^{*1.5}/d_h^*}{\left[1 + \frac{13.95\varphi}{u^{*0.5} d_h^*} + \left(\frac{13.766}{K}\right)^3 \frac{1}{u^{*1.5} d_h^{*3}}\right]^{1/3}} + K_1 u^{*2} = 1 \tag{27}$$

It can be seen from this equation that, the dimensionless velocity u^* depends only on d_h^* for a given shape of the cross-section.

Besides u^*, we need the dimensionless temperature θ to get q_A^* from Eq. (8). The dimensionless temperature θ is to be calculated from

$$\theta = \exp(-4Nu\, z) \tag{28}$$

where z is the dimensionless axial coordinate and Nu is Nussselt number. z is defined as follows:

$$z = x^*/Pr \tag{29}$$

in which Pr is Prandtl number.

In ducts of arbitrary cross-sectional area, for developed flow and developing thermal conditions, Yilmaz and Cihan [7] have given the following equation for Nusselt number Nu_∞:

$$Pr \to \infty: Nu_\infty = Nu_{\infty,\infty} + \frac{1.615\Phi/(z/\varphi)^{1/3}}{\left[1 + 1.88\left(\frac{zNu^3_{\infty,\infty}}{\varphi\Phi^3}\right) + 3.93\left(\frac{zNu^3_{\infty,\infty}}{\varphi\Phi^3}\right)^{4/3}\right]^{1/2}}$$

where

$$Nu_{\infty,\infty} = 3.657\left[1 + (1 - 1/n)\left(0.5155\frac{d^{*^2}}{3 - d^*} - 1\right) + \Delta\Phi\right] \tag{31}$$

$$\Delta\Phi = \Delta\Phi_{\max}\frac{0.95(n-1)^{0.5}}{1 + 0.038(n-1)^3} \tag{32}$$

$$\Delta\Phi_{\max} = \frac{7 \times 10^{-3} d^{*8}}{(1 + 10 d^{*-28})(1 + 64 \times 10^{-8} d^{*28})^{0.5}} \tag{33}$$

$$\Phi = 1 + \frac{\left[3(d^*/2)^{7/8}/(1 + d^*) - 1\right]}{1 + 0.25/(n-1)} \tag{34}$$

For developing flow conditions and $z \to 0$, the shape of the duct has no influence on Nusselt number and Nusselt number Nu_o is well described by the following equation:

$$\begin{matrix} z \to 0 \\ Pr \neq \infty \end{matrix} : Nu_o = \frac{0.6774 z^{-0.5}}{f Pr^{1/6}} \tag{35}$$

The function f is dependent on Prandtl number and can be formulated using the data of Gauler [8] and Merk [9] as follows:

$$f = \left(1 + \frac{0.105}{Pr + \sqrt{Pr}/3} + \frac{0.0468}{Pr}\right)^{1/6} \tag{36}$$

The following equation for developing flow and developing thermal condition can be used:

$$Nu = \left(Nu_\infty^4 + Nu_o^4\right)^{1/4} \tag{37}$$

This equation is compared with the nearly exact equation given by Shome and Jensen [10] for simultaneously developing flow and heat transfer in circular tubes for different Prandtl numbers ($Pr = 0.1 - \infty$) and the maximum differences are found as $-1.8/5.1\%$. For other shaped ducts, Eq. (37) produces good results too [11].

3. Equations for long ($d_h^* \to 0$) and short ducts ($d_h^* \to \infty$)

In the following sections, velocity, Nusselt number and the amount of heat transferred for two limiting cases (long and short ducts) are considered.

3.1. Velocity u^*

For long ducts ($d_h^* \to 0$) it follows from Eq. (27):

$$d_h^* \to 0: u^* = \frac{d_h^{*2}}{64\varphi} \tag{38}$$

This equation means that frictional pressure loss prevails for long ducts. For short ducts we should distinguish between two cases. For the ideal case, there is no contraction and expansion losses, so that $K_l = 0$. For this case, Eq. (27) yields

$$\begin{matrix} d_h^* \to \infty \\ K_l = 0 \end{matrix} : u^* = 0.1741 d_h^{*2/3} \tag{39}$$

In the case of $K_l \neq 0$, it follows:

$$\begin{matrix} d_h^* \to \infty \\ K_l \neq 0 \end{matrix} : u^* = K_l^{-1/2} \tag{40}$$

3.2. Nusselt number

For Nusselt number, we can again distinguish between two cases. Eq. (35) can be used directly for developing flow conditions ($Pr \neq \infty$):

$$\begin{matrix} d_h^* \to \infty \\ Pr \neq \infty \end{matrix} : Nu = \frac{0.6774 z^{-0.5}}{f Pr^{1/6}} \tag{41}$$

For developed flow conditions, Eq. (30) yields

$$\begin{matrix} d_h^* \to \infty \\ Pr \to \infty \end{matrix} : Nu = 1.615\Phi\varphi^{1/3} z^{-1/3} \tag{42}$$

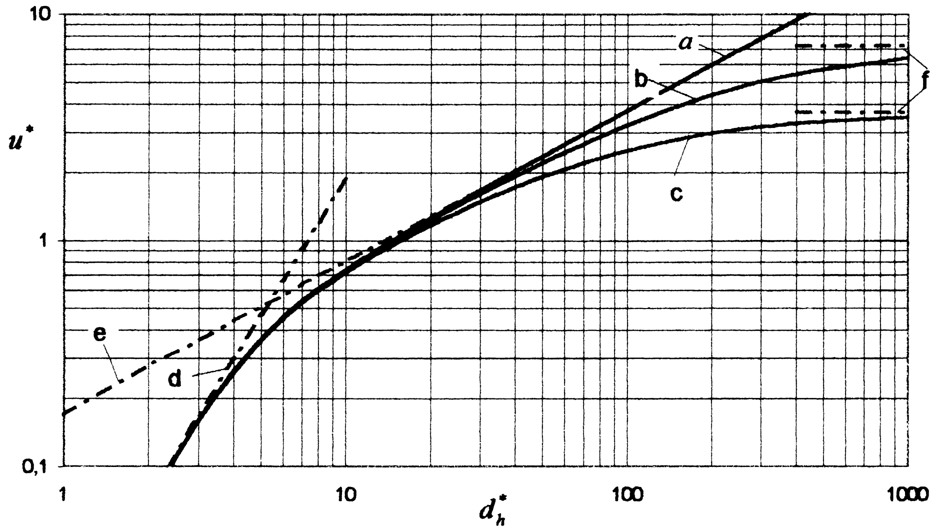

Fig. 2. Variation of dimensionless velocity u^* with dimensionless hydraulic diameter d_h^* for different values of porosity ε. (a) $\varepsilon = 1$; (b) $\varepsilon = 0.9$; (c) $\varepsilon = 0.8$; (d) Eq. (38); (e) Eq. (39); (f) Eq. (40).

3.3. The amount of heat transferred

3.3.1. Long ducts

In long ducts one can assume that $\theta \to 0$ and, therefore, Eq. (43) can be obtained from Eqs. (8) and (38):

$$d_h^* \to 0 : q_A^* = 1.5625 \times 10^{-2} d_h^{*2} / \varphi \tag{43}$$

3.3.2. Short ducts

For short ducts, the mean dimensionless temperature θ can be obtained from Eq. (28) as

$$z \to 0 : \theta = 1 - 4Nu\, z \tag{44}$$

Introducing Eq. (44) into Eq. (8), Eq. (45) can be obtained for the amount of heat transferred:

$$z \to 0 : q_A^* = 4u^* Nu\, z \tag{45}$$

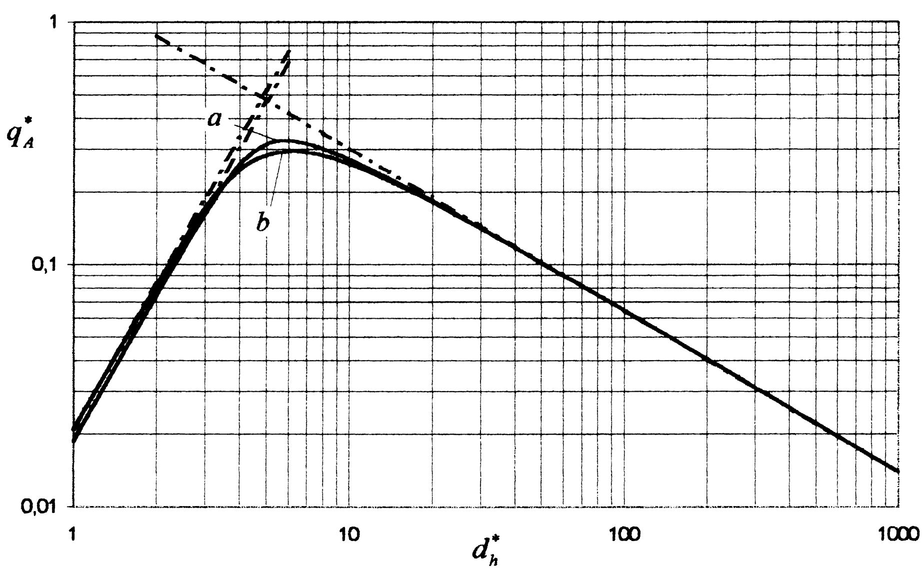

Fig. 3. Variation of dimensionless heat transfer per cross-sectional area q_A^* with dimensionless hydraulic diameter d_h^* for (a) equilateral triangular duct ($n = 1.654$) and (b) triangular duct with $n \to \infty$ for $K_1 = 0$ and $Pr = 0.7$.

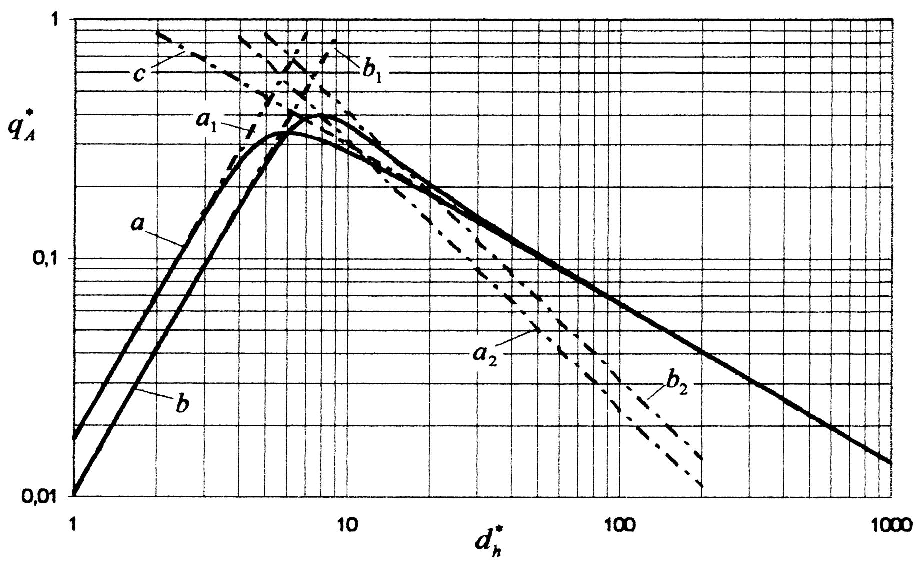

Fig. 4. Variation of dimensionless heat transfer per cross-sectional area q_A^* with dimensionless hydraulic diameter d_h^* for square duct (a), and parallel plate duct (b) for $K_l = 0$ and $Pr = 0.7$. a_1: Eq. (49); a_2: Eq. (52); b_1: Eq. (50); b_2: Eq. (53); c: Eq. (51).

Using Eqs. (39), (41) and (42), one gets the following equations for the practically important case $K_l = 0$:

$$\begin{aligned} &d_h^* \to \infty \\ &Pr \neq \infty : q_A^* = \frac{1.1306}{fPr^{2/3}} d_h^{*-2/3} \\ &K_l = 0 \end{aligned} \tag{46}$$

$$\begin{aligned} &d_h^* \to \infty \\ &Pr \to \infty : q_A^* = 3.607 \frac{\varphi^{1/3}\Phi}{Pr^{2/3}} d_h^{*-10/9} \\ &K_l = 0 \end{aligned} \tag{47}$$

$$\begin{aligned} &d_h^* \to \infty \\ &Pr \to 0 \ : q_A^* = \frac{1.883}{Pr^{0.5}} d_h^{*-2/3} \\ &K_l = 0 \end{aligned} \tag{48}$$

4. Discussion of the results

In Fig. 2, dimensionless velocity u^* is given as a function of dimensionless hydraulic diameter d_h^* for equilateral triangular duct for various values of porosity ε. The curves a, b and c are for $\varepsilon = 1{,}0.9$ and 0.8, respectively. The limiting curve d for $d_h^* \to 0$ is independent of ε according to Eq. (38). The other limiting curve e is valid for $\varepsilon = 1$ and $d_h^* \to \infty$. This curve is obtained from Eq. (39). The limiting curves f calculated using Eq. (40) for $d_h^* \to \infty$ and $\varepsilon \neq 1$ are horizontal lines, because the frictional part of the pressure loss can be neglected.

Variation of dimensionless heat transfer per cross-sectional area q_A^* with dimensionless hydraulic diameter d_h^* is illustrated in Fig. 3 for two different triangular ducts for $Pr = 0.7$ and $K_l = 0$. Curve a represents the equilateral triangular duct($n = 1.654$) and curve b represents the triangular duct with $n \to \infty$. It is clear from Fig. 3 that, there is no substantial difference between various triangular ducts. The asymptotic curves shown on the figure are calculated from Eqs. (43) and (46).

In Fig. 4, q_A^* is given as a function of d_h^* for rectangular ducts for $Pr = 0.7$ and $K_l = 0$. Curve a is valid for a square duct ($d^* = 1$, $n = 1.273$) and curve b is valid for a parallel plate duct ($d^* = 2$, $n = \infty$). As can be seen from the figure, the maximum value of q^* for a parallel plate duct is greater than that for a square duct. Curves a_1 and b_1 seen in Fig. 4, are asymptotic curves for $d_h^* \to 0$ for square and parallel plate ducts, respectively. They are calculated using Eq. (43):

$$a_1 : q_A^* = 1.762 \times 10^{-2} d_h^{*2} \tag{49}$$

$$b_1 : q_A^* = 1.042 \times 10^{-2} d_h^{*2} \tag{50}$$

Curve c is given for the short duct asymptotic case ($d_h^* \to \infty$) according to Eq. (46) for $Pr = 0.7$ and $K_l = 0$:

$$c : q_A^* = 1.396 d_h^* {-2/3} \tag{51}$$

This curve does not depend on the duct shape, and therefore, it is valid for both square and parallel plate ducts.

For the middle values of d_h^* ($5 \leqslant d_h^* \leqslant 25$), the curves

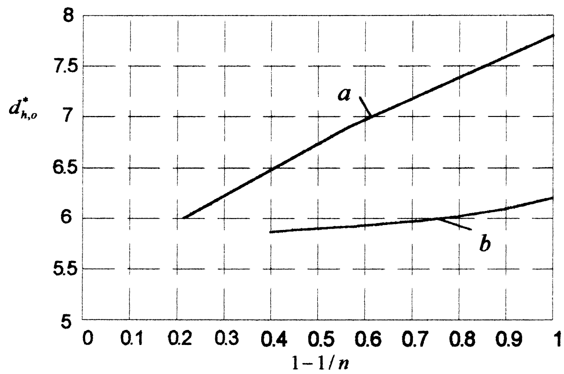

Fig. 5. Variation of optimum dimensionless hydraulic diameter $d^*_{\mathrm{h,o}}$ with the parameter $1-1/n$ for rectangular duct (a) and triangular duct (b) for $Pr=0.7$ and $K_{\mathrm{l}}=0$.

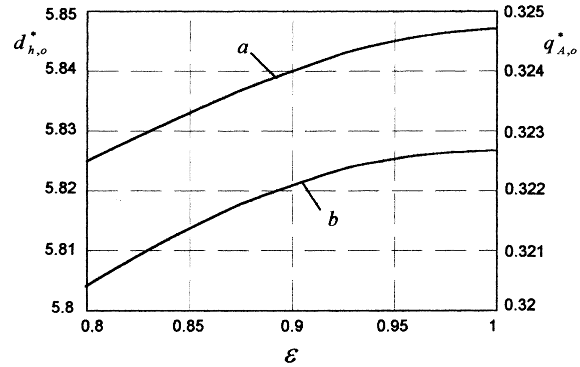

Fig. 7. Variation of $d^*_{\mathrm{h,o}}$ and $q^*_{A,\mathrm{o}}$ with porosity ε for equilateral triangular duct for $Pr=0.7$ (a: $d^*_{\mathrm{h,o}}$; b: $q^*_{A,\mathrm{o}}$).

a and b first approach the corresponding limiting curves a_2 and b_2 which are valid for $Pr\rightarrow\infty$. These curves are calculated from Eq. (47):

$$a_2{:}q^*_A = 3.977 d_{\mathrm{h}}^{*-10/9} \tag{52}$$

$$b_2{:}q^*_A = 5.237 d_{\mathrm{h}}^{*-10/9} \tag{53}$$

After this region, curves a and b approach the limiting curve c which represents the case for $Pr=0.7$. Therefore, one can see easily why the maximum value of q^*_A ($q^*_{A,\mathrm{o}}$) for the parallel plate duct is greater than that for the square duct.

In Figs. 5 and 6, the optimum values of dimensionless hydraulic diameter d^*_{h} ($d^*_{\mathrm{h,o}}$) at which dimensionless heat transfer per cross-sectional area q^*_A has a maximum value of $q^*_{A,\mathrm{o}}$ and $q^*_{A,\mathrm{o}}$ are given, respectively.

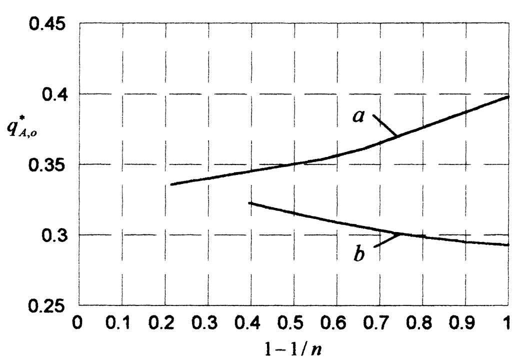

Fig. 6. Variation of maximum dimensionless heat transfer per cross-sectional area $q^*_{A,\mathrm{o}}$ with the parameter $1-1/n$ for rectangular duct (a) and triangular duct (b) for $Pr=0.7$ and $K_{\mathrm{l}}=0$.

For triangular ducts, $d^*_{\mathrm{h,o}}$ does not change significantly with n, and $q^*_{A,\mathrm{o}}$ decreases slightly with increasing n, as expected. In the case of rectangular ducts, $d^*_{\mathrm{h,o}}$ and $q^*_{A,\mathrm{o}}$ increase with n increasing and the reason for it is explained in the discussion of Fig. 4.

The influence of ε on $d^*_{\mathrm{h,o}}$ and $q^*_{A,\mathrm{o}}$ for an equilateral triangular duct is given in Fig. 7 for $Pr=0.7$. Decreasing the value of ε results in increased pressure loss because of contraction and expansion. Therefore, velocity and heat transfer will decrease. However, for some practical applications, such as rotary regenerators and plate heat exchangers, the dependence of velocity and heat transfer on ε can be ignored.

The values for $d^*_{\mathrm{h,o}}$ and $q^*_{A,\mathrm{o}}$ are given in Table 1 for equilateral triangular, square, circular and parallel plate ducts for different Pr numbers.

In Fig. 8, influence of Pr number on $d^*_{\mathrm{h,o}}$ and $q^*_{A,\mathrm{o}}$ for an equilateral triangular duct is shown. As it can be seen from this figure, $d^*_{\mathrm{h,o}}$ and $q^*_{A,\mathrm{o}}$ both decrease with the increase of Pr number. $d^*_{\mathrm{h,o}}$ can be described with a maximum deviation of $\mp 5\%$ for all Pr numbers and all shapes of ducts (including the examples given in Table 1) by Eq. (54):

$$d^*_{\mathrm{h,o}} = 6.0\varphi^{3/8} Pr^{-1/4}\left[1+0.01/Pr^2\right]^{1/16} \tag{54}$$

Using $d^*_{\mathrm{h,o}}$ obtained from Eq. (54) one can get u^* iteratively from Eq. (27) and calculate z, and then Nu and θ from Eqs. (29), (37) and (28), respectively. Knowing these values $q^*_{A,\mathrm{o}}$ can be determined from Eq. (8) almost exactly, because $q^*_{A,\mathrm{o}}$ is not very sensitive to d^*_{h} around $d^*_{\mathrm{h,o}}$. The following equation can be used to get approximate values of $q^*_{A,\mathrm{o}}$ for all Pr numbers and all shapes of ducts:

$$q^*_{A,\mathrm{o}} = \frac{0.335\varphi^{1/4} Pr^{-1/2}}{\left(1+0.47\varphi^{-1/3}/Pr\right)^{1/4}} \tag{55}$$

Table 1
Optimum d_h^* values ($d_{h,o}^*$) and maximum values of q_A^* ($q_{A,o}^*$) for different ducts and Pr numbers for $K_l = 0$

Duct	Pr	0.1	0.7	1	7	10	50	100
Equilateral triangular	$d_{h,o}^*$	10.460	5.850	5.37	3.425	3.175	2.135	1.778
	$q_{A,o}^*$	0.6192	0.3227	0.2809	0.119	0.1006	0.0457	0.0323
Square	$d_{h,o}^*$	10.575	6.000	5.512	3.500	3.225	2.187	1.837
	$q_{A,o}^*$	0.6413	0.3355	0.2918	0.123	0.1039	0.0471	0.0334
Circular	$d_{h,o}^*$	11.125	6.437	5.912	3.812	3.500	2.375	1.987
	$q_{A,o}^*$	0.6684	0.3490	0.3033	0.128	0.1077	0.0489	0.0346
Parallel plate	$d_{h,o}^*$	13.137	7.825	7.162	4.462	4.087	2.737	2.312
	$q_{A,o}^*$	0.7935	0.3980	0.3421	0.138	0.1161	0.0523	0.0370

Table 2
Comparison between the values obtained in this work, calculated numerically and by Eqs. (54) and (55) and the values given by Bejan and Sciubba

	δ					q'^*				
Pr	0.72	6	20	100	1000	0.72	6	20	100	1000
This work (numerical)	6.016	6.102	6.121	6.148	6.124	0.472	0.516	0.522	0.524	0.524
Eqs. (54) and (55)	5.881	5.874	5.874	5.874	5.874	0.468	0.516	0.522	0.524	0.524
Bejan and Sciubba [1]	6.066	6.155	6.156	6.110	6.050	0.479	0.522	0.527	0.526	0.523

This equation reproduces the numerical values with an accuracy of $\mp 5\%$. From the last two equations, it can be considered that $d_{h,o}^*$ and $q_{A,o}^*$ are functions of only Prandtl number Pr and the shape factor φ.

In the work of Bejan and Sciubba [1], optimum values δ and maximum values q'^* are given for a parallel plate duct. The following relationships are valid between these parameters and the dimensionless parameters used in the present work:

$$\delta = \frac{d_h^*}{2^{1/4}} Pr^{1/4} \tag{56}$$

$$q'^* = 2^{1/2} q_A^* Pr^{1/2} \tag{57}$$

The values of δ and q'^* for parallel plates given by Bejan and Sciubba are compared with the values determined from Eqs. (54) and (55) and numerical values obtained in this work in Table 2.

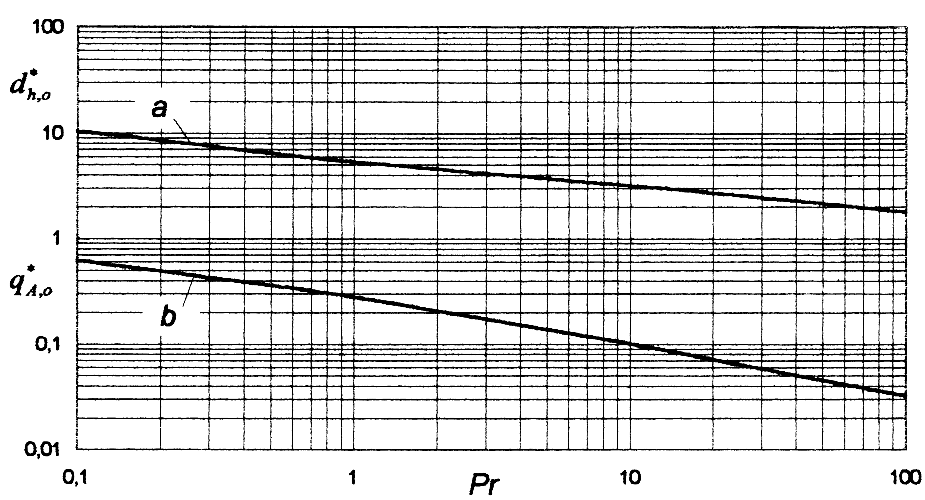

Fig. 8. Variation of $d_{h,o}^*$ and $q_{A,o}^*$ with Pr for equilateral triangular duct for $K_l = 0$ (a: $d_{h,o}^*$; b: $q_{A,o}^*$).

For the given range of *Pr* numbers, the values given by Bejan and Sciubba are in good agreement with the values calculated from Eqs. (54) and (55). However, it is not possible to make a comparison for ducts of other shapes, since there is no available study.

5. Conclusions

The result of this investigation is that for a given pressure loss and for a certain Prandtl number, both maximum dimensionless heat flux $q^*_{A,\mathrm{o}}$ and optimum dimensionless hydraulic diameter $d^*_{\mathrm{h,o}}$ increase with increasing values of the duct shape factor φ. With the equations derived one can easily determine the optimum dimensions of ducts of arbitrary cross-sectional area.

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